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## NUMERICAL INVESTIGATION OF CHAOS IN THE TIME-DELAY

IKEDA LASER-RING CAVITY.

by

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Instabilities and chaos in a ring cavity -laser system containing a non-linear medium is investigated in detail. The system is modelled using the delay-difference equations obtained by Ikeda<sup>1</sup>:

$$E(t) = A + B * E(t - \tau_r) * \exp(i[\psi(t) - \psi_0]) \quad \dots\dots\dots(1)$$

$$\gamma^{-1} d\psi/dt = -\psi(t) + |E(t - \tau_r)|^2 \quad \dots\dots\dots(2)$$

where A is proportional to the amplitude of the incident field, B is a dissipation parameter defined by  $B = R \exp(-\alpha L/2)$ ,  $\alpha$  is the linear absorption rate. L is the length of the non-linear medium, E is a dimensionless complex electric field inside the cavity,  $\psi$  is the phase shift due to the medium.  $\gamma^{-1}$  is the inversion relaxation time of the medium,  $\psi_0$  is the mistuning of the medium of the cavity and  $\tau_r$  the round trip time.

It is assumed that there is significant detuning between the atomic and light frequencies so that the dispersive limit applies. Le

Berre et. al.<sup>2</sup> has solved similar ring cavity equations which allowed for both the dispersive and absorptive limits.

Equations (1) and (2) were numerically integrated with the parameters  $B$  and  $\psi_0$  set at 0.4 and 0.0 respectively. Figures 1-3 display the numerical computations, for three different values of  $A$ , after transients have decayed. Each figure displays  $\text{Re}E$  versus  $\text{Im}E$  and  $\text{Re}E$  versus  $\psi_0$ , for three values of  $\gamma t_r$ .

The 2d mapping has a period doubling route to chaos as  $A$  is increased. Equation (1) and (2) also have a period doubling route when  $\gamma t_r$  is greater than 500. At low values of  $\gamma t_r$  there is no period doubling as apparent from figure 3, where <sup>there</sup> is an attractor at  $\gamma t_r=10$  and there is period 16 at  $\gamma t_r=1000$ . For all three values of  $A$  there is a smooth change from the attractor present at low values of  $\gamma t_r$ , to the corresponding mapping attractor which manifests itself at  $\gamma t_r > 500$  approximately. In figure 2 is the attractor at  $\gamma t_r > 500$  fundamentally the same as the 2d mapping attractor for the same value of  $A$ ? Le Berre et. al. have suggested that these two attractors are different because for the delay-differential equations (1) and (2) the dimension, as evaluated from the Kaplan-Yorke conjecture, goes to finity as  $\gamma t_r$  does; whereas the dimension of the Ikeda-mapping attractor is less than 2.

We have computed the dimension of the attractor for  $\gamma t_r$  in the interval [70,1500] and found the fractal dimension to be finite and less than 2. This proves that the two attractors are fundamentally the same, thus validating the use of the adiabatic approximation. We use the method of Grassberger and Procaccia to obtain the correlation exponent  $\nu$ . It has been proven that  $\nu$  is a lower bound on the Hausdorff dimension<sup>3</sup>. In practice  $\nu$  is obtained from the integral correlation function<sup>4</sup>,

$$C_d(1) = \lim_{N \rightarrow \infty} \frac{1}{N^2} [\text{number of pairs } (n,m) \text{ with } (\sum_{i=1}^d |x_{n+i} - x_{m+i}|^2)^{1/2} < 1] \dots \dots (3)$$

A time series was constructed from the real part of the electric field, denoted by  $[x_i]_{i=1}^N$ . In evaluating  $C_d(1)$  we only used successive points separated by the round trip time  $t_r$ . Figures 4 and 5 display plots of  $\ln C_d(1)$  versus  $\ln l$  for a range of embedding dimensions  $d$ . Figure 4 is for the mapping with  $A=3.9$  and figure 5 is for equations (1) and (2) with  $\gamma t_r=200$  and  $A=3.9$ . The correlation index  $\nu$  was

computed over the linear region and in table 1 we exhibit  $\nu$  for various values of  $\gamma t_r$ .

Table 1

Estimates of the correlation exponent  $\nu$  for Ikeda's equations with  $A=3.9$ ,  $B=0.4$ ,  $\psi_0=0.0$ .

$t_r$	$\nu$
2d map	1.34 +/- 0.06
1500	1.45 +/- 0.30
500	1.31 +/- 0.08
200	1.31 +/- 0.15
100	1.35 +/- 0.09
70	1.37 +/- 0.08

In conclusion we have found that when  $\gamma t_r$  is greater than 500 (approximately) we have period doubling and that the attractor is fundamentally the same as the Ikeda mapping attractor. This implies that in the adiabatic limit the physics is essentially unchanged i.e. the adiabatic approximation is a valid approximation.

#### Acknowledgements

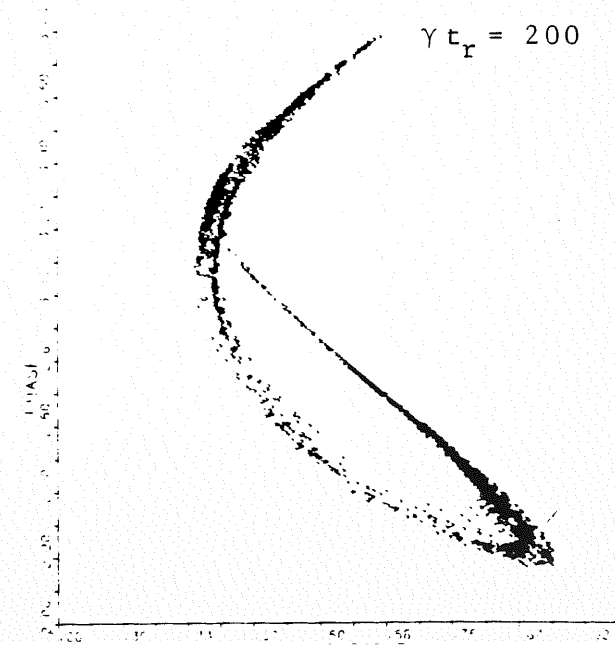
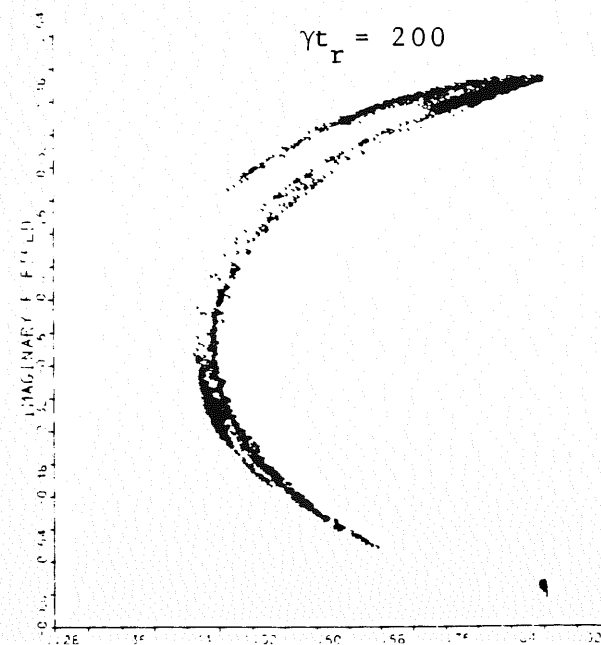
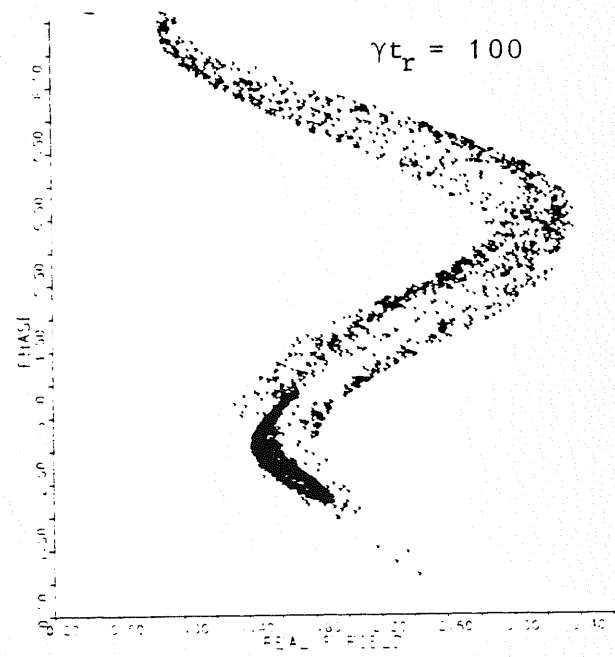
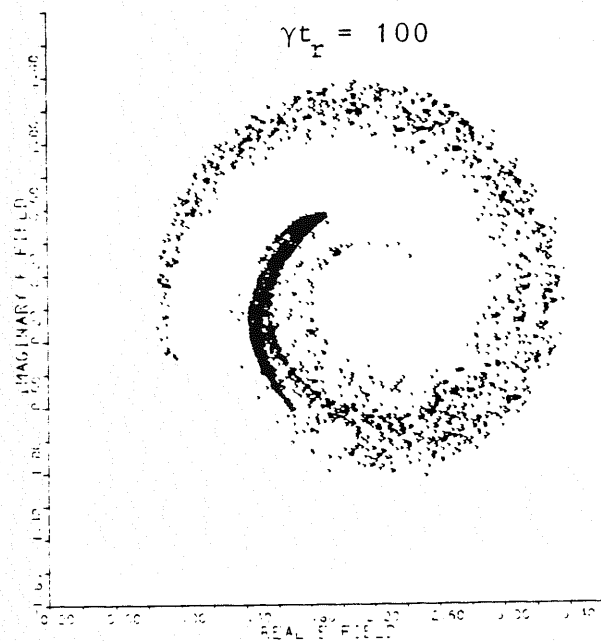
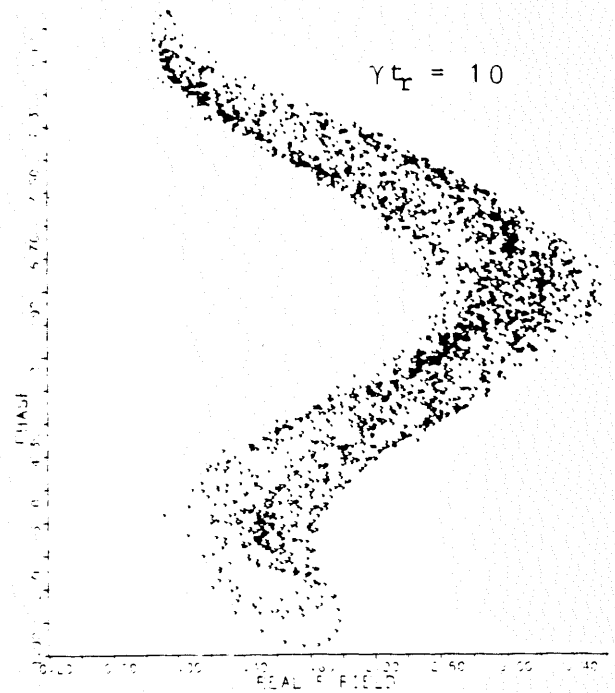
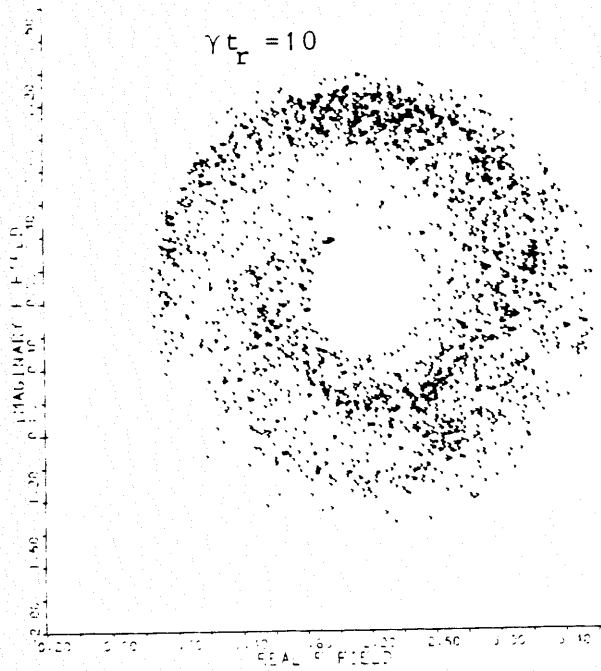
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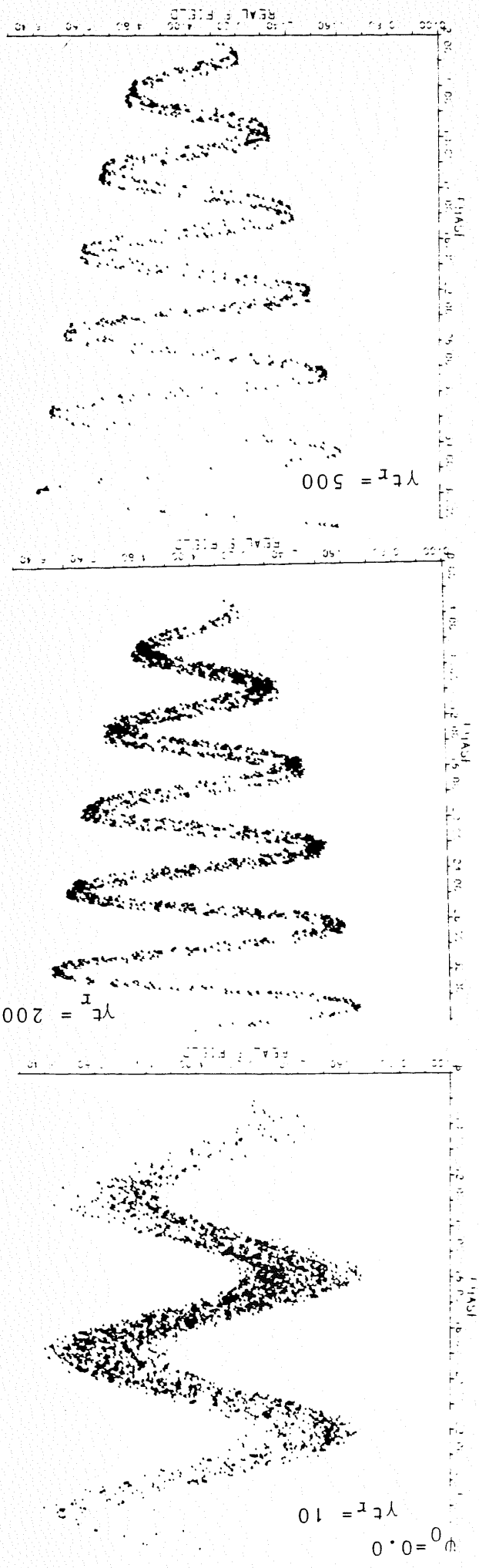
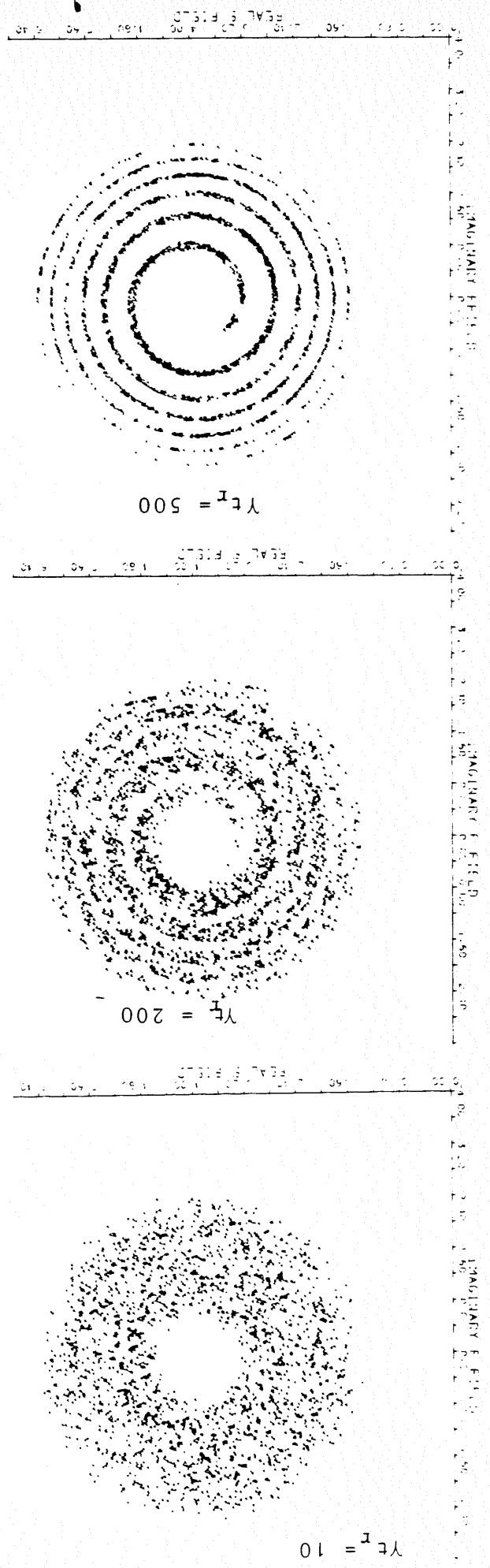
#### References

- (1) K. Ikeda, H. Daido and O. Akimoto, Phys. Rev. Lett. 45, 709 (1980).
- (2) M. Le Berre, E. Ressayre, A. Tallet and H. M. Gibbs, Phys. Rev. Lett. 56, 274 (1986).
- (3) P. Grassberger and I. Procaccia, Physica 9D, 189 (1983).
- (4) P. Grassberger and I. Procaccia, Phys. Rev. A 28, 2591 (1983).

Figure 1

$A=2.155$   $B=0.4$   $\psi_0=0.0$





$A = 3.9$   $B = 0.4$

Figure 2

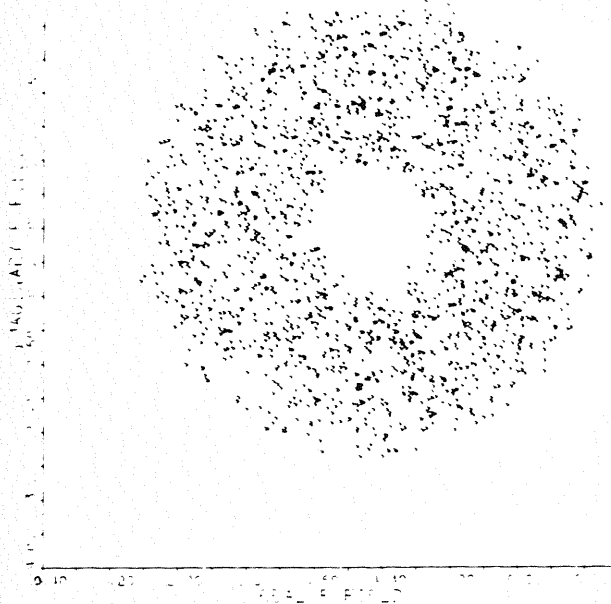
Figure 3

$A=4.265$

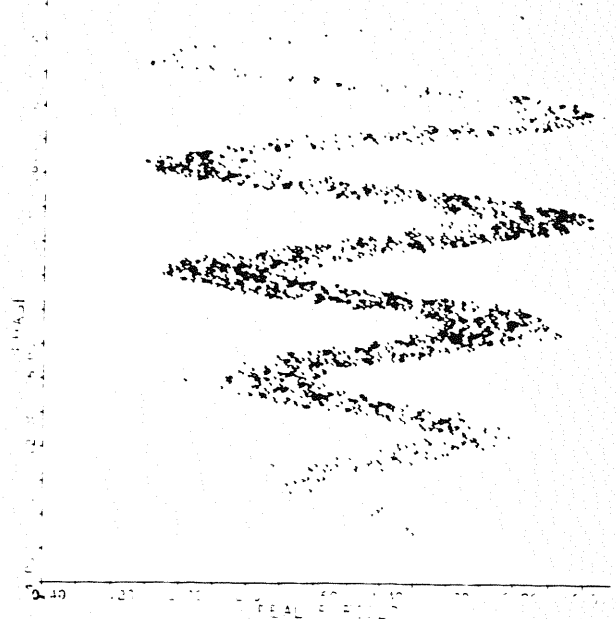
$B=0.4$

$\psi_0=0.0$

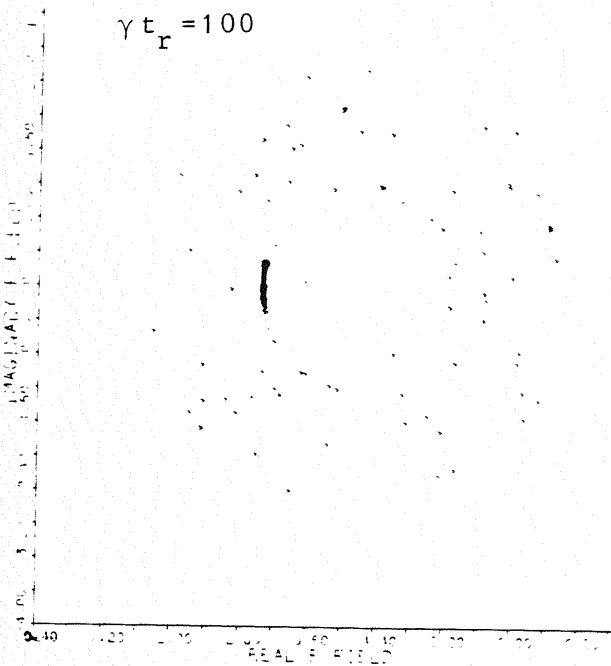
$\gamma t_r = 10$



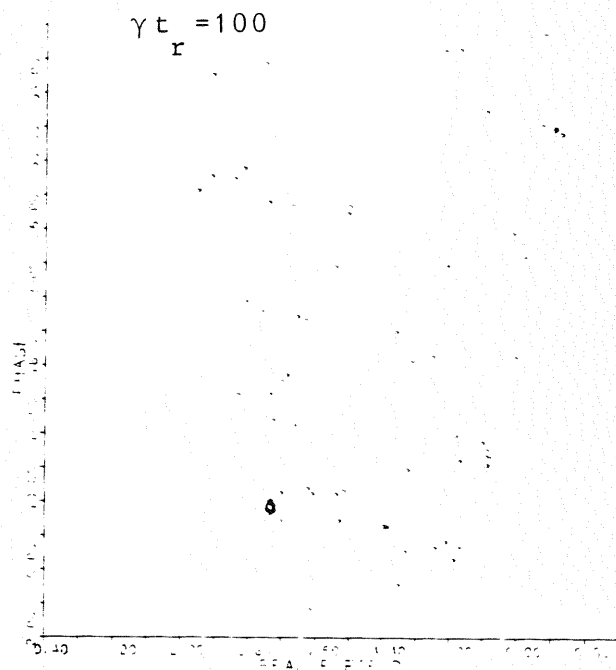
$\gamma t_r = 10$



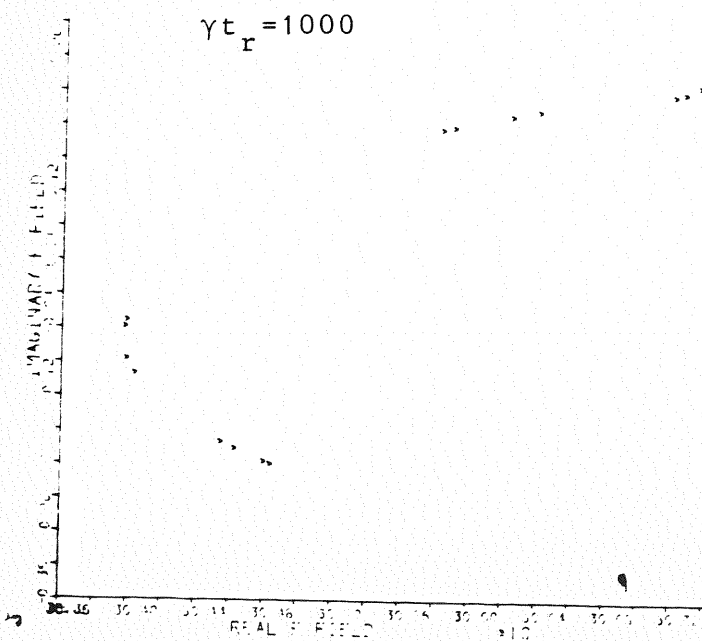
$\gamma t_r = 100$



$\gamma t_r = 100$



$\gamma t_r = 1000$



$\gamma t_r = 1000$

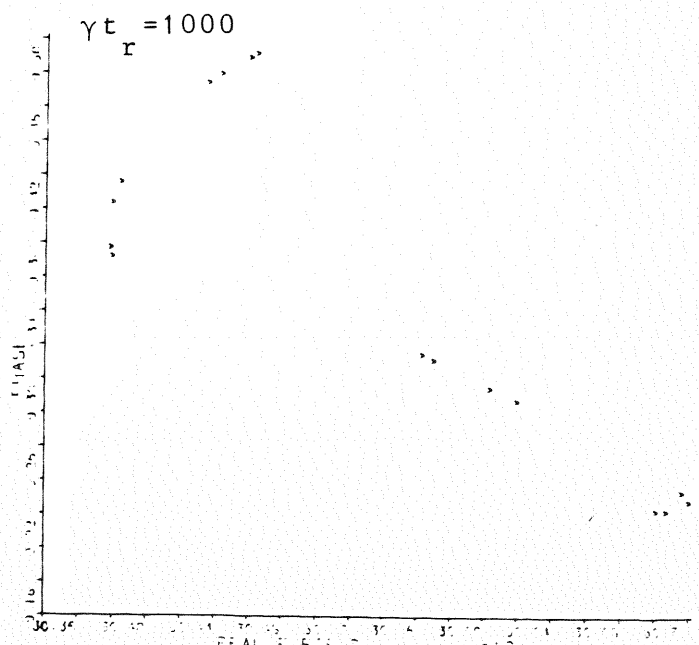


Figure 4

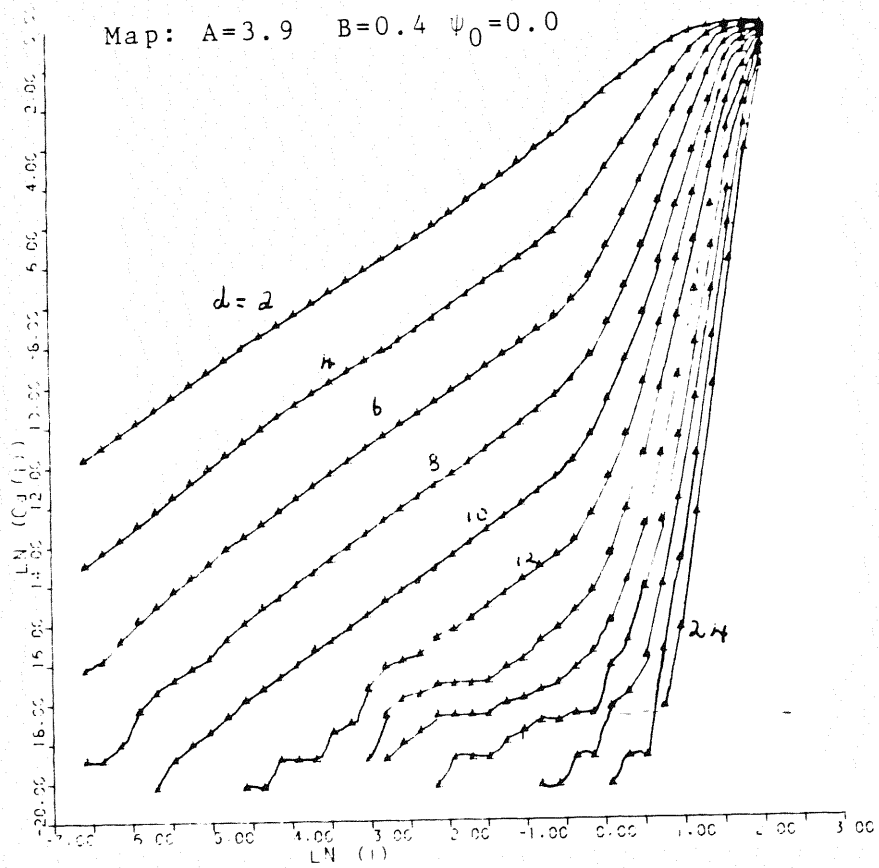


Figure 5

